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Fourier transform and efficiency of calculations of one currency pair to the second

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Abstract

The article gives the solution of the Gelfand-Levitan equation for exchange rates, using the Fourier transform. The results are used to simulate the behavior of exchange rate of currency pairs.

Keywords

Exchange rates forecasting, Gelfand-Levitan equation, Fourier transform, mathematical modeling.

Introduction

Consideration of the exchange rate in economic science in the development of the global monetary system has been constantly taken on new meaning. If we consider the exchange rate as an economic category, it is the price of the currency in a certain country. But every country does not exist by itself, especially if it is a country with a market-oriented economy. In modern conditions, the scope of an indicator of the exchange rate is very broad: it is used in the state statistical calculations, determine the country's position in the world economy, reflects the close relationship between export and import of goods and services, serves as a criterion evaluating the effectiveness of foreign economic activity and the level of production and, as a consequence, tool for counting the net external financial position of the nation, etc.

In consideration of the foregoing, the importance of the exchange rate can be more aptly described as an indicator of the price of currency of one country expressed in the currency of another country.

Actual scientific problem, which predetermines the research objective of this article, implies the development of tools to determine the value and to model exchange rates. The study, analysis and systematization of different approaches and attitudes about the exchange rate from the perspective of early and modern scientific research allowed to estimate the contribution of scientists to the future considerations of exchange rates in the system of economic development of the country and to highlight the most interesting and relevant works, which formed the basis for the formation and development of scientific solutions of the author in this article[1].

Main points

In order to form the mathematical apparatus of modeling the behavior of currency pairs, the author suggests the analogy of a physical process of elastic waves propagation and economic processes[2]. The similarities of currency

1 Balatskii, E.V., Serebrennikova, A.V. (2008), "New tool imperatives in exchange rate modeling" ["Novye instrumental'nye imperativy v modelirovanii valyutnykh kursov"], *Vestnik Moskovskogo universiteta. Seriya "Ekonomika"*, No. 5, pp. 15-24; Bunkina, M.K. (2004), *Money. Banks. Currency: Study guide* [*Den'gi. Banki. Valyuta: Uchebnoe posobie*], DIS, Moscow, 175 p.; Grigor'ev, K.A. (2008), "The methodical base of the formation of exchange rate in the conditions of world economy globalization", *Bulletin of the ENGECON. Economics series. Issue 3 (22)* ["Metodicheskie osnovy formirovaniya valyutnogo kursa v usloviyakh globalizatsii mirovoi ekonomiki", *Vestnik INZhEKONA. Ser. Ekonomika, Vyp. 3 (22)*], SPbGIEU, St. Petersburg, pp. 21-28; Krasavina, L.N., "Currency problems of innovative development of Russian economy" ["Valyutnye problemy innovatsionnogo razvitiya ekonomiki Rossii"], *Den'gi i kredit*, available at: www.cbr.ru/publ/main; Panilov, M.A. (2009), "The development of theory of exchange rate and evolution of principles of its modelling" ["Razvitie teorii valyutnogo kursa i evolyutsiya printsipov ego modelirovaniya"], *Audit i finansovyi analiz*, No. 4, pp. 261-284.

2 Alekseev, A.S., Dobrinskii, V.I. (1975), "Some questions of practical use of inverse dynamical problems of seismicity", *Mathematical problems of geophysics. Issue 6, Part 2* ["Nekotorye voprosy prakticheskogo ispol'zovaniya obratnykh dinamicheskikh zadach seismiki", *Matematicheskie problemy geofiziki. Vyp. 6, Ch. 2*], VTs SO AN SSSR, Novosibirsk, pp. 7-53; Vladimirov, V.S. (1981), *The equations of mathematical physics* [*Uravneniya matematicheskoi fiziki*], Nauka, Moscow,

movement with the results with the results of seismic studies were noted when considering a large number of dependencies, namely, characterized by rapid change in exchange rate in unstable times and rapid changes in the density of medium (rock mass) in case of solid mass fluctuations. Author also noted that the case of failed rocks (the same for exchange rates) is characterized by an increased value in some areas of average squared difference and the decrease in the square of the correlation coefficient in case of trend line plotting.

In connection with the foregoing reveals the scientific task of finding the dependence of exchange rate of one currency pair from the other, which decision is proposed by the author to develop through the proof of the applicability of the Gelfand-Levitan equation for exchange rates. For this purpose, the requirements to incoming functions are formulated, ambiguity in the notations are eliminated, restrictions for the model are introduced.

Mathematical modeling allows predicting the dynamics of the second currency pair at first one, both in the long and short terms.

This article shows how to solve this equation for further instrumental application.

Let the recorded exchange rate $f(t)\epsilon C^2$ belongs to the class of functions with continuous second derivative and the opposite to the determined rate $\varphi(t)\epsilon C^1$ – to the class of functions with continuous first derivative, then, if

$$\frac{\partial f}{\partial \varphi} \leq F,$$

that is limited, then these functions satisfy the integral equation of Gelfand-Levitan with the initial condition:

$$\frac{\partial f}{\partial \varphi} = c(+0)\partial(t),$$

$c(+0)=[2\varphi(0,0)]^{-1}$, where $c(+0)=-f(+0)$.

The problem is connected with the equation of Gelfand-Levitan

$$D_f^x\varphi(x,t) \equiv -1,\ |t|<x, \quad (1)$$

$$D_f^x\varphi(x,t) \equiv -2f(+0)\varphi(x,t) - -\int_{-x}^{x}\varphi(x,s)\tilde{f}'(t-s)ds, \quad (2)$$

where $\tilde{f}'(t)$ – a derivative of an even extension of the function $\tilde{f}'^{+}(t)$ defined in the points of the domain of existence $(t\neq 0)$.

For

$$T>0\,(x\epsilon(0,t),\ x<t<2T-x). \quad (3)$$

The problem (25) is equivalent, provided that:

$$(D_f^x\varphi,\ \varphi)>0,\ \forall\,\varphi\epsilon L_2(-x,x),\ \forall\,x\epsilon(0,T),\ \varphi\neq 0. \quad (4)$$

512 p.; Tikhonov, A.N., Goncharskii, A.V., Stepanov, V.V., Yagola, A.G. (1990), *Numerical methods of the incorrect problems solution* [*Chislennye metody resheniya nekorrektnykh zadach*], Nauka, Moscow, 232 p.

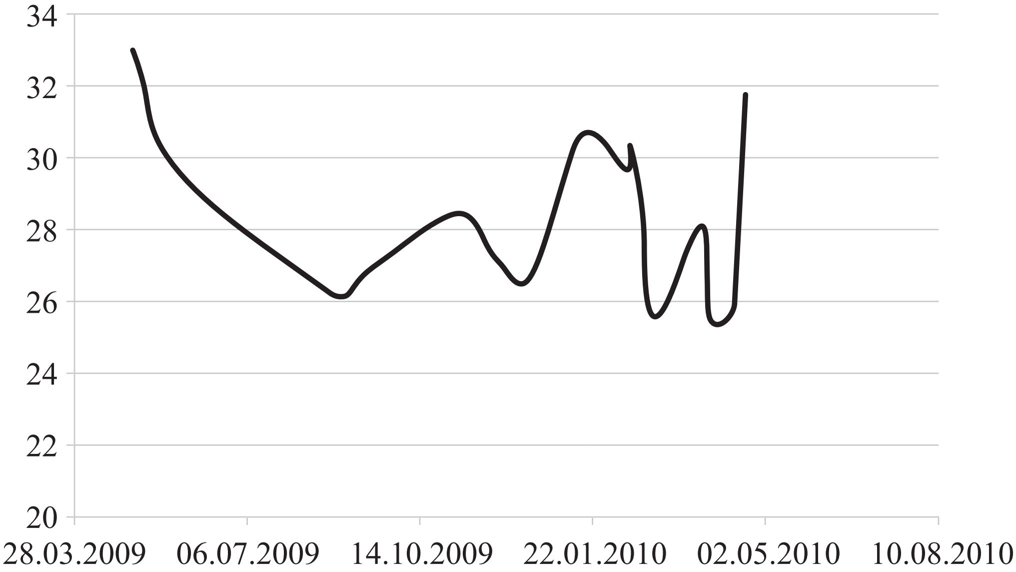

Fig. 1. Testing the use of the Fourier transform method for the currency pair EUR \ USD as of 2010

That is, the solution of the inverse problem (1-3) subject to (4) exists, uniquely and is associated with either the solution of equation:

$$c(+0) = [2\varphi(0,0)]^{-1} \quad (5)$$

$$c(x) = \varphi(0,0)[2\varphi^2(x, x-0)]^{-1},$$

$$x \in [0, T] \quad (6)$$

or can be obtained explicitly in a linear-fractional rate transformation

$$f_1 \cong \frac{\varepsilon(X) - \varepsilon(0)}{d - c} \quad (7)$$

Unidimensional inverse problem has been investigated[3]. However, we have used a different numerical scheme for solving the problem. The problem was solved by the Fourier transform of the equation (1-2):

$$F[\varphi(x_i, t)] + \frac{1}{2f(+0)} F[\varphi(x_i, t)] \cdot$$

$$\cdot F[\tilde{f}'(t)] = F[\frac{1}{2f(+0)}] \quad (8)$$

Where

$$F[\varphi(x_i, t)] = \frac{F[\frac{1}{2f(+0)}]}{(1 + \frac{1}{2f(+0)} F[\tilde{f}'(t)])} \quad (9)$$

Calculated Russian Ruble to US Dollar Rate

After applying an inverse Fourier transform, it was obtained:

$$\varphi(x_i, t) = F^{-1}\{\frac{F[\frac{1}{2f(+0)}]}{(1 + \frac{1}{2f(+0)} F[\tilde{f}'(t)])}\} \quad (10)$$

3 Gel'fand, I.M., Levitan, B.M. (1951), "On the definition of differential equation on its spectral function" ["Ob opredelenii differentsial'nogo uravneniya po ego spektral'noi funktsii"], *Izv. AN SSSR. Ser. Mat.*, Vol. 15, No. 4, pp. 309-360.

Taking into account that:

$$F[\tilde{f}'(t)](\varpi) = \int_{-\infty}^{+\infty} \tilde{f}'(t) e^{-i\varpi t} dt =$$

$$= \tilde{f}(t) e^{-i\varpi t}|_{-\infty}^{\infty} - (-i\varpi) \int_{-\infty}^{+\infty} \tilde{f}(t) e^{-i\varpi t} dt =$$

$$= i\varpi\, F[\tilde{f}(t)](\varpi)$$

was obtained:

$$\varphi(x_i, t) = F^{-1}\{\frac{F[\frac{1}{2f(+0)}](\varpi)}{1 + \frac{i\varpi}{2f(+0)} F[\tilde{f}(t)](\varpi)}\} \quad (11)$$

Next $\varphi(x_i\, t)$ we need to place in the Form (9), where $t = x_{i-1}$.

Here, the discrete Fourier transform implies

$$\tilde{f}_m = \sum_{k=0}^{n-1} f_k e^{-i\varpi_m X_k} = \sum_{k=0}^{n-1} f_k e^{-i2\pi(mk/n)} \quad (12)$$

wherein $m = 0, 1, \ldots; n-1$; $\omega_m = m\Delta\omega$, $\Delta\omega = 2\pi/T$; and under the discrete inverse Fourier transform:

$$f_k = \frac{1}{n} \sum_{m=0}^{n-1} \tilde{f}_m e^{i\varpi_m X_k} \quad (13)$$

where $k = 0, 1, \ldots; n-1$; and a convolution of the Fourier transform [10]:

$$\sum_{k=0}^{n-1} (\sum_{j=0}^{n-1} K_{k-j} z_j \Delta x) e^{-i\varpi_m X_k} = \Delta x \tilde{z}_m \tilde{k}_m \quad (14)$$

When testing the method on the currency pair EUR \ USD as of 2010 (Fig. 1), the error does not exceed the long-term forecast of 6-10% with a probability belief 0.95.

From the point of view of computational mathematics and mathematical physics, the solutions to one-dimensional Gelfand-Levitan equations, being a parametric family of integral equations of convolution type, are developed enough. The method of Fourier transform is very effective for the same equations of convolution type, but subtlety here lies in choosing the regularization parameter and the conditions of uniqueness of the problem solution.

Conclusion

Modern science has accumulated a large amount of theoretical material and practical experience of economic-mathematical modeling, forecasting, including in the context of exchange rate problems. Practical application of the leading theories of exchange rates at the current stage of economic development in Russia, along with the mathematical apparatus similar to that used in this paper will help to solve important problems that predict the needs of sustainable economic growth, stability of the national currency, the welfare of the population.

The article convincingly proved similarity of the results of currency movement with the results of seismic studies, noted that in the case of failed rocks (the same for exchange rates) is characterized by an increased value in some areas of average squared difference and the decrease in the square of the correlation coefficient in case of trend line plotting. On this basis, the dependence of exchange rate changes of one currency pair from the other has been determined by the proof of the applicability of the Gelfand-Levitan equation for the exchange rates.

Proposed in this article, the scientific solution to the problem of modeling the behavior of the currency pairs allows creating short-term and long-term forecasts of behavior of exchange rates based on the following initial data:

1) time step;

2) initial value of the regularization parameter, which is then elaborated by the solution of the generalized disparity of Tikhonov by Newton's method;

3) estimated faulty proportion of the integral equation right side imposition;

4) initial value of the2nd calculated exchange rate;

5) area of search of the problem solution for the 2nd exchange rate.

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Преобразования Фурье и эффективность вычисления одной валютной пары по второй

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Аннотация

В статье приведено решение уравнения Гельфанда-Левитана для валютных курсов, с использованием преобразования Фурье. Результаты применяются для моделирования поведения курса валютных пар.

Ключевые слова

Прогнозирование валютных курсов, уравнение Гельфанда-Левитана, преобразование Фурье, математическое моделирование.

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